Quantity, Risk, and Return

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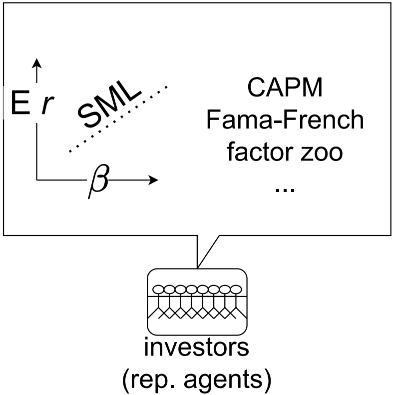
Research question

Expected stock returns: why different stocks earn different returns?

- Theoretical answer: risk!
  investors are risk averse, require compensation for bearing risk
  $\Rightarrow$ high-risk high-return

- Empirical challenges:
  - high-risk high-return is elusive in data (e.g., flat SML)
  - risk-based models hardly predict stock returns
    vs. unstructured predictions: machine learning + characteristic

Integrate quantity into factor pricing



New elements to factor pricing framework:

- not all investors think about risk-return trade-offs and factor pricing
- those who do are affected by trading with others

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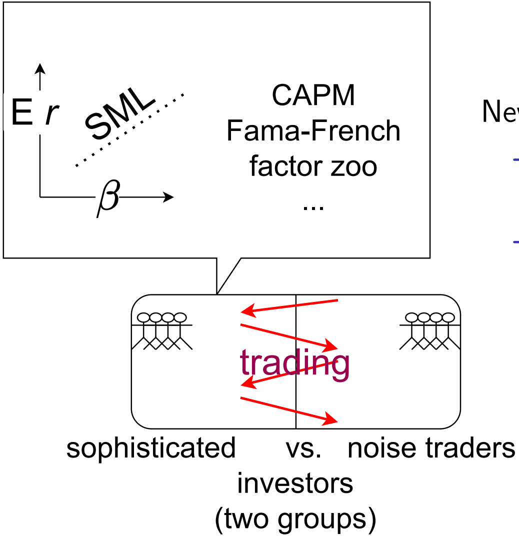

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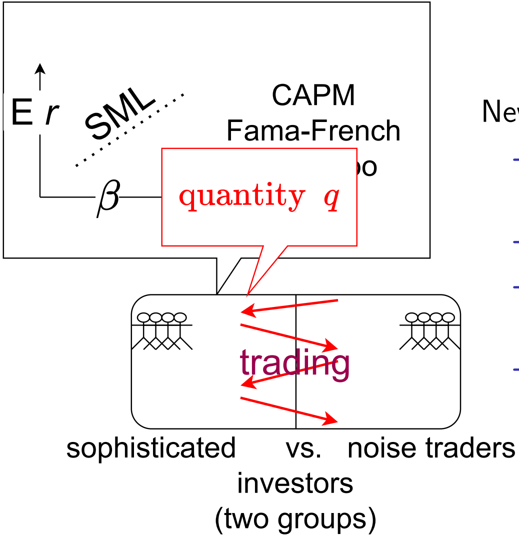
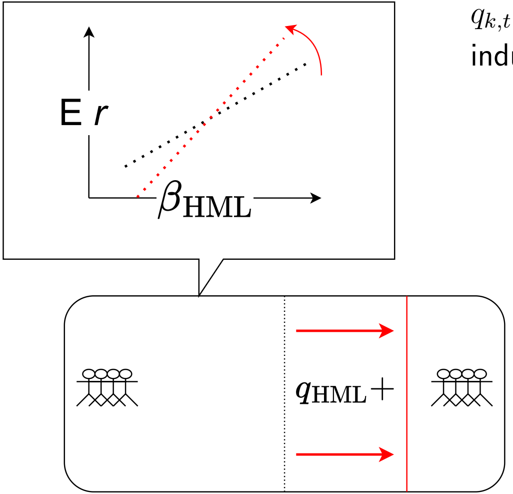


New elements to factor pricing framework:

- not all investors think about risk-return trade-offs and factor pricing
- those who do are affected by trading with others
- their holding quantity of factor risks (q) fluctuates accordingly
- q in turn affects their required compensation for bearing risks

How does q enter risk-return modeling?



sophisticated investors vs. noise traders

$q_{k,t}$: sophisticated investors' factor risk holdings induced by trading flows

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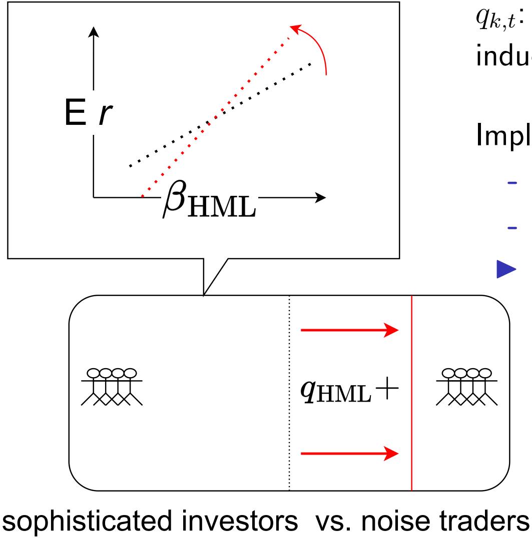


$q_{k,t}$: sophisticated investors' factor risk holdings induced by trading flows

Implication: expected stock returns depend

- not only on factor loadings β,
- but also importantly on q time series

- together, "beta times q" (BTQ) as predictor of stock returns

Results preview and outline

- Finding 1: Risk-return trade-off depends on quantity
- Finding 2: BTQ predicts stock returns
- Finding 3: BTQ selects a few fundamental factors from the “factor zoo”
- Alternative model “quantity only, no risk” does not work

Construct $q_{k,t}$: quantity of factor risk induced by noise trading flow

Step 1: Stock-level flows: mutual fund flow-induced trading (FIT)

- following Coval and Stafford (2007) and Lou (2012), with S12 data

$$\$\text{flow}_{i,t}^{\text{stock}} = -\sum_{\text{fund } m} \underset{\substack{\uparrow \\ \text{fund flow}}}{\$\text{flow}_{m,t}^{\text{fund}}} \times \underset{\substack{\uparrow \\ \text{fund's port. weight}}}{\text{weight}_{m,i,\text{quarter}-2}^{\text{fund}}}$$

(positive $\$\text{flow}_{i,t}^{\text{stock}}$ means sophisticated investors buy)

Construct $q_{k,t}$: quantity of factor risk induced by noise trading flow
Key step: aggregate stock-level flow to factor-level

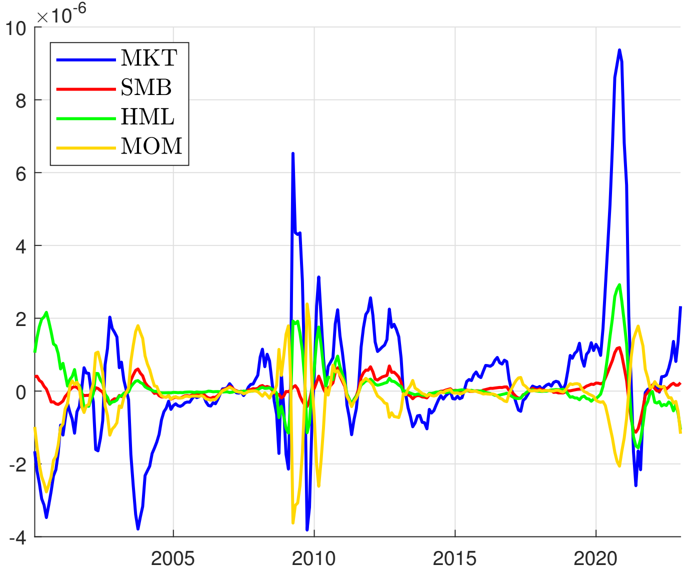
- Aggregate stock-level flow to factor-level

$$\text{flow}_{k,t}^{\text{factor}} := \sum_{\text{stock } i} \$\text{flow}_{i,t}^{\text{stock}} \, \underset{\substack{\uparrow \\ \text{stock's exposure to factor } k}}{\text{cov}_{i,k,t}}$$

- Accumulate flow shocks in past six months
  - additionally, normalize by total market cap and standardize time series

construction details

$q_{k,t}$ time-series plot (before standardization)



summary stats

→ **Finding 1: Risk-return trade-off depends on quantity**
(simple non-parametric evidence that q matters for expected returns)

- Finding 2: BTQ predicts stock returns

- Finding 3: BTQ selects a few fundamental factors from the “factor zoo”

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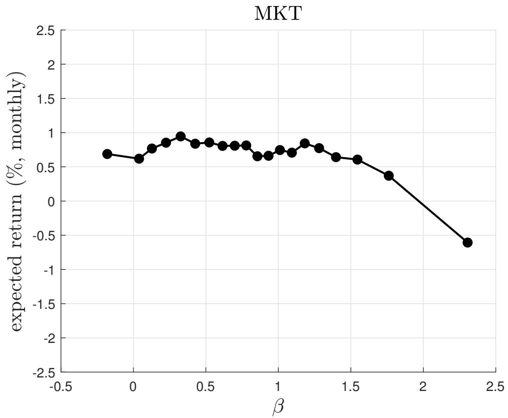
Security Market Lines (SML) is flat

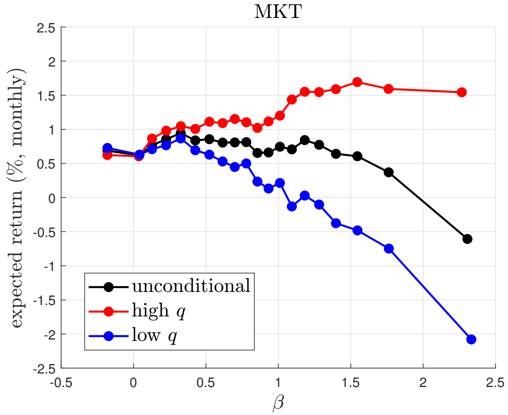
contradicts "high risk, high return"



SML as non-parametric regression: $\mathbb{E}_t[r_{i,t+1}] = Er(\beta_{i,k,t})$ for the stock-month panel

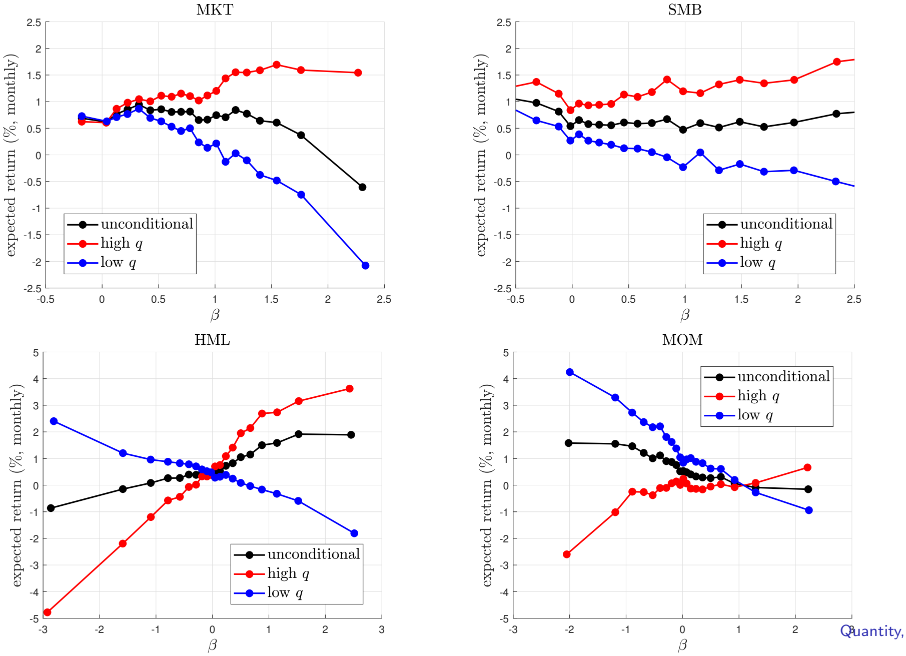
Upgraded SML varies with q

"high risk, high return" conditional on high q



SML as non-parametric regression: $\mathbb{E}_t[r_{i,t+1}] = Er(\beta_{i,k,t})$

upgraded SML: one more input: $\mathbb{E}_t[r_{i,t+1}] = Er(\beta_{i,k,t}, q_{k,t})$


MKT
SMB
HML
MOM
expected return (%, monthly)
β
unconditional
high q
low q

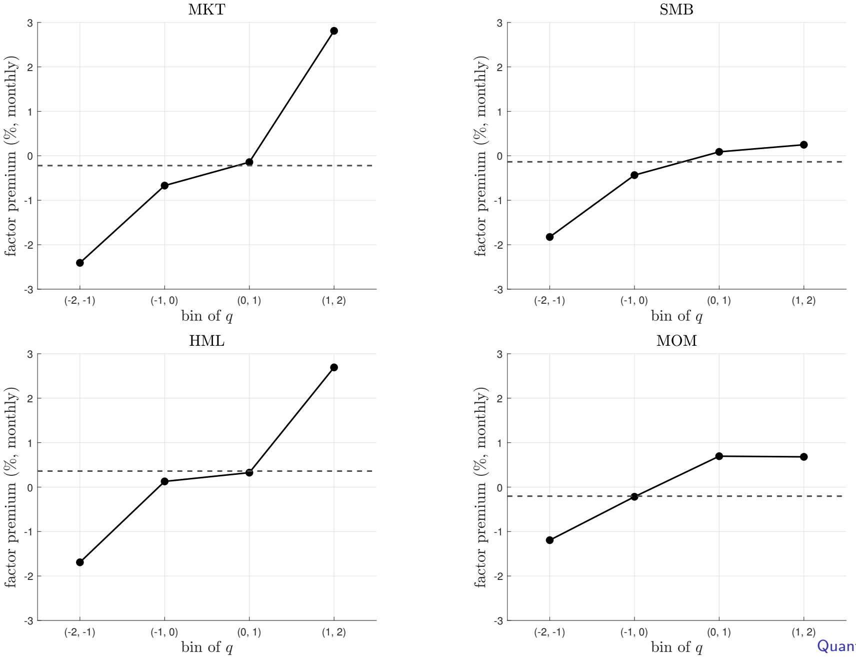
Fama-MacBeth factor premium increases with $q_{k,t}$

Estimation:

- Fama-MacBeth: cross-sectional reg $r_{i,t+1}$ on $\widehat{\beta}_{k,i,t}$, get coef. $\gamma_{k,t+1}$
  - Canonical: $\mu_k =$ time-series average of $\gamma_{k,t+1}$
  - Upgraded: varying $\mu_{k,t} = \mu_k(q_{k,t})$ conditional on $q_{k,t}$

Model:

$$\mathbb{E}_t[r_{i,t+1}] = Er(\beta_{i,k,t}, q_{k,t}) = \beta_{i,k,t}\mu_k(q_{k,t})$$


MKT
SMB
HML
MOM
factor premium (%, monthly)
bin of q
(-2, -1)
(-1, 0)
(0, 1)
(1, 2)
3
2
1
0
-1
-2
-3

- Finding 1: Risk-return trade-off depends on quantity
  (simple non-parametric evidence that q matters for expected returns)

→ **Finding 2: BTQ predicts stock returns**
  (risk-based explanation for expected stock returns)

- Finding 3: BTQ selects a few fundamental factors from the "factor zoo"

- Alternative model "quantity only, no risk" does not work

BTQ: a stock return prediction model

- Factor premium is constant vs. linear function of $q_{k,t}$:

$$\mu_{k,t} = \mu_k \quad \text{vs.} \quad \lambda_k q_{k,t}$$

- Factor pricing (APT):

$$\mathbb{E}_t[r_{i,t+1}] = \sum_k \beta_{i,k} \mu_{k,t}$$

- Plug in:

$$\mathbb{E}_t[r_{i,t+1}] = \sum_k \mu_k \beta_{i,k,t} \quad \text{vs.} \quad \sum_k \lambda_k \beta_{i,k,t} q_{k,t}$$

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- Estimation: BTQ predictive regression (stock-month panel)

$$r_{i,t+1} = \sum_k \lambda_k \left(\widehat{\beta}_{i,k,t} q_{k,t}\right) + error_{i,t+1}, \qquad \forall i,t$$

vs. "β-only"

$$r_{i,t+1} = \sum_k \mu_k \widehat{\beta}_{i,k,t} + error_{i,t+1}, \qquad \forall i,t$$

BTQ vs. β-only, single factor

	Fama-French-Carhart factors				Across 153 JKP factors		
	MKT	SMB	HML	MOM	Q25	Median	Q75
Panel A: IS R^2 comparison, full sample 2000-2022 (%)							
BTQ	1.01	0.30	1.00	0.91	0.39	0.62	0.95
β-only	0.05	0.05	0.12	0.06	0.02	0.06	0.10
Panel B: OOS R^2 comparison, evaluation window 2010-2022 (%)							
BTQ	0.75	0.60	0.84	0.65	0.20	0.38	0.67
β-only	0.05	-0.10	0.15	0.02	-0.03	0.04	0.11

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BTQ vs. β-only, single factor, coefficients

	Fama-French-Carhart factors				Across 153 JKP factors		
	MKT	SMB	HML	MOM	Q25	Median	Q75
Panel C: full-sample coefficient comparison: 2000-2022							
BTQ							
λ_k	1.80	0.72	1.48	1.77	0.62	0.99	1.48
t-stat	(4.18)	(2.76)	(3.52)	(3.38)	(2.24)	(2.96)	(3.69)
β-only							
μ_k	0.38	0.31	0.56	-0.50	-0.33	-0.14	0.22
t-stat	(1.07)	(1.25)	(1.71)	(-1.23)	(-1.52)	(-0.71)	(1.11)

BTQ vs. β-only, multi-factor

	CAPM	FF3	FF3C	FF5	FF5C
	$K = 1$	3	4	5	6
Panel A: IS R^2, full sample 2000-2022 (%)					
BTQ	1.01	1.17	1.19	1.17	1.21
β-only	0.05	0.17	0.21	0.18	0.22
Panel B: OOS R^2, evaluation window 2010-2022 (%)					
BTQ	0.75	1.03	1.07	0.44	0.65
β-only	0.05	0.15	0.22	-0.26	-0.05

coefficents

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$\rightarrow$ **Finding 3: BTQ selects a few fundamental factors from the "factor zoo"**
  (new perspective to the factor zoo problem with quantity)

- Alternative model "quantity only, no risk" does not work

“Taming the factor zoo” with BTQ

- So many proposed factors, which are fundamental?

- New perspective to discipline factors with quantity
  - BTQ vs. β-only prediction models:

$$r_{i,t+1} \sim \sum_k \lambda_k \left(\widehat{\beta}_{i,k,t} q_{k,t}\right) \text{ vs. } \sum_k \mu_k \widehat{\beta}_{i,k,t}$$

  - old question: $\mu_k > 0$? is there factor premium?
  - new question: $\lambda_k > 0$? does factor premium vary with investor risk holdings?

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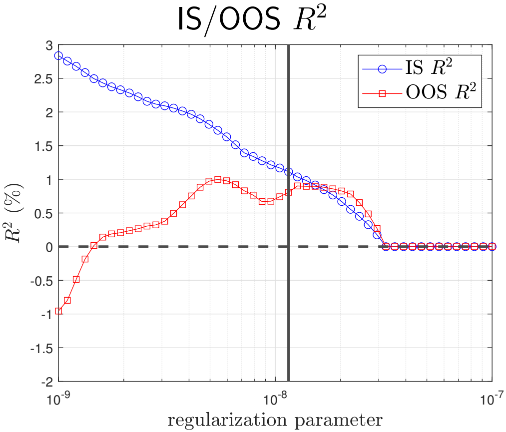
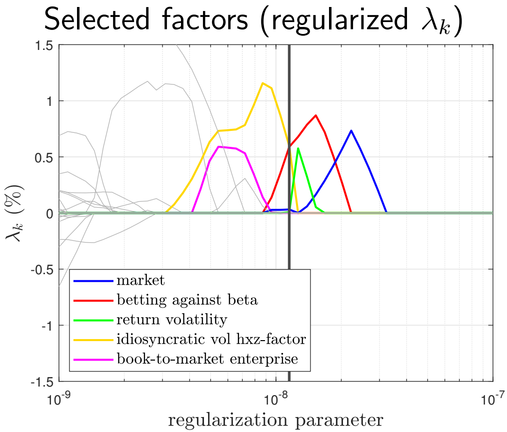
- Implementation
  - overload BTQ prediction model with 159 FF+JKP factors
  - run BTQ as high-dimensional prediction, select factors with Lasso
  - alternatively, extract PCs from the 159 factor zoo, all else equal

BTQ, selecting from factor zoo

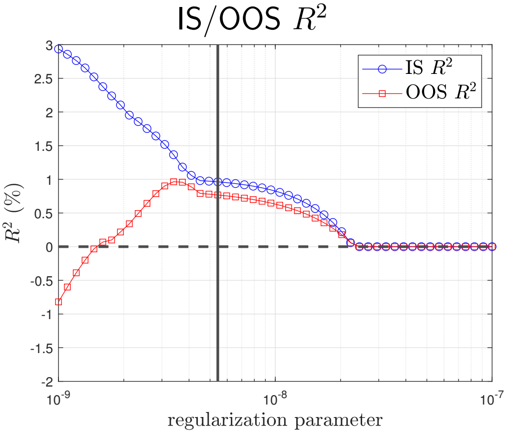


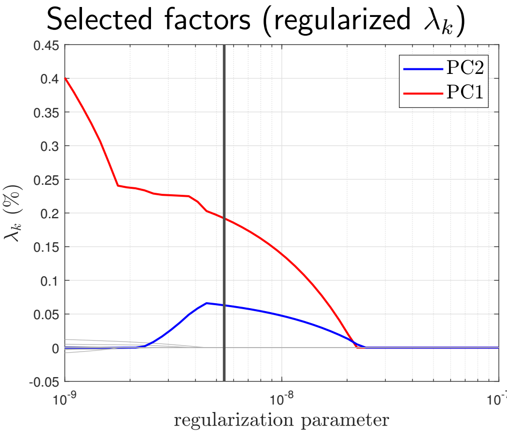


▸ selected factors

▸ β-only

BTQ, selecting from PC factors





▸ PC details

▸ β-only

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$\rightarrow$ **Alternative model "quantity only, no risk" does not work**
(factor structure is essential according to alternative comparison)

Alternative model: “Quantity-only”

- So far, BTQ vs. β-only $\Rightarrow q$ is critical
- Is risk necessary? or is quantity alone sufficient?

Alternative model: "Quantity-only"

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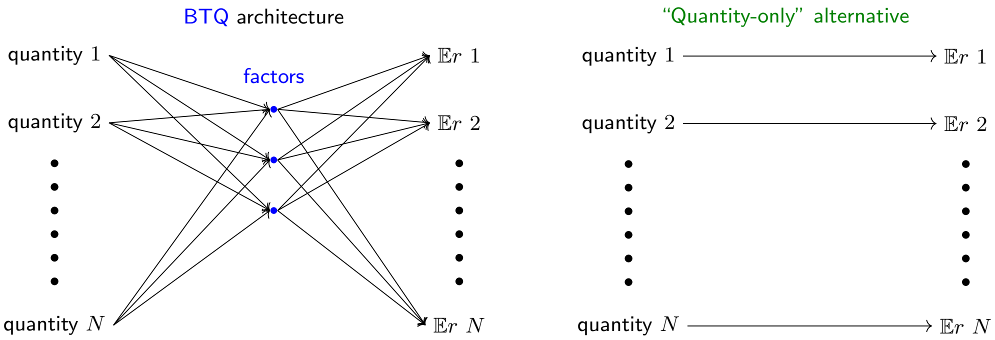

- "Quantity-only" bypasses factors and APT, directly linking stock-level q to $\mathbb{E}r$
- BTQ: aversion to factor risks vs. Quantity-only: "aversion" to stocks
  macro vs. micro elasticities (Gabaix and Koijen, 2022)

"Quantity-only" alternative model does not forecast stock returns

- "Quantity-only": stock return prediction with stock-level q

$$r_{i,t+1} = \lambda_i^{\text{stock}} q_{i,t}^{\text{stock},h} + error_{i,t+1},$$

$$\text{where } q_{i,t}^{\text{stock},h} := \frac{1}{h} \sum_{h'=0}^{h-1} \$\text{flow}_{i,t-h'}^{\text{stock}} / \text{market cap}_{i,t-1-h'}$$

	A. constant λ^{stock}				B. λ^{stock} by size quintile		C. λ_i^{stock} by stock	
h	IS R^2(%)	OOS R^2(%)	λ^{stock}	t-stat	IS R^2(%)	OOS R^2(%)	IS R^2(%)	OOS R^2(%)
1	0.000	0.000	-0.10	-0.27	0.003	-0.001	0.47	-232
3	0.000	0.000	0.17	0.29	0.003	0.000	0.39	-155
6	0.005	0.006	0.75	1.01	0.007	0.006	0.41	-107
9	0.004	0.004	0.75	0.99	0.006	0.006	0.38	-96
12	0.003	-0.007	0.77	1.01	0.006	-0.004	0.38	-81

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- "Quantity-only": stock return prediction with stock-level q

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6	0.005	0.006	0.75	1.01	0.007	0.006	0.41	-107
9	0.004	0.004	0.75	0.99	0.006	0.006	0.38	-96
12	0.003	-0.007	0.77	1.01	0.006	-0.004	0.38	-81

- Implications:
  - quantity works through factors to determine $\mathbb{E}r$
  - cross-sectional no-arb is effective (Kozak, Nagel, and Santosh, 2020)

Conclusion

factor pricing + quantity to explain expected stock returns

Channel:

- noise trading $\rightarrow$ factor risk quantity $\rightarrow$ factor premium

Findings:

- Risk-return tradeoff (β-$\mathbb{E}r$ relation) depends on quantity
- BTQ predicts stock returns
- A new perspective to the "factor zoo" problem with quantity
  - Factor pricing condition is essential, stock-level quantity alone is not sufficient

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