Quantity, Risk, and Return

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AFA Annual Meetings
January 5, 2026, Philadelphia

Research question

Expected stock returns: why different stocks earn different returns?

- In theory: risk
  investors are risk averse, require compensation for bearing risk
  $\Rightarrow$ high-risk high-return

- Empirical challenges:
  - high-risk high-return is elusive in data (e.g., flat SML)
  - risk-based models (β) hardly predict stock returns
    vs. machine learning + characteristics: unstructured predictions

What is missing in factor pricing?

Integrate quantity into risk-return modeling

- APT: expected stock return driven by factor exposures (β)

$$\mathbb{E}_t r_{i,t+1} = \sum_k \mu_{k,t} \beta_{i,k,t}$$

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- Add quantity ($q_{k,t}$, factor-level time series)

  - model: $$\mu_{k,t} = \lambda_k q_{k,t}$$

  - $q \uparrow$: sophisticated investors buying factor risk recently
    constructed as retail selling via mutual fund flow-induced trading (FIT)

  - finding: strong q-μ positive association (for almost all factors)

  - interpretation: hold more quantity $\Rightarrow$ greater risk compensation

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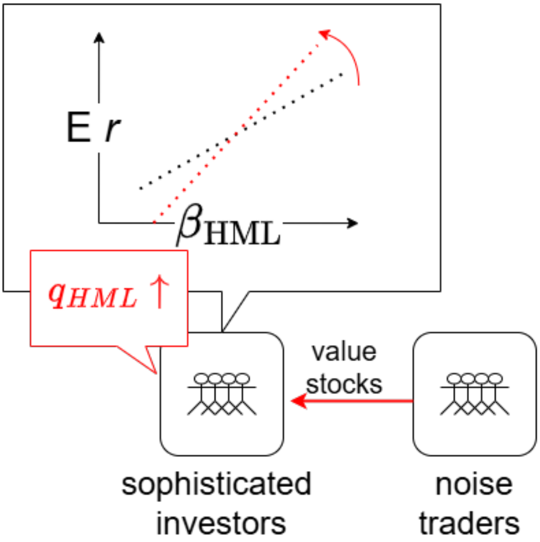
Together:

- "β-times-quantity" (BTQ) predicts stock returns (OOS $R^2 \approx 1\% \gtrapprox$ ML sota)

$$r_{i,t+1} \sim \beta_{i,k,t} q_{k,t} \quad \text{vs. canonical} \quad r_{i,t+1} \sim \beta_{i,k,t}$$



Expected stock return $\mathbb{E}_t r_{i,t+1}$ depends on:

- not only factor loading $\beta_{i,k,t}$,
- but also $q_{k,t}$

Construct $q_{k,t}$

the quantity of factor risk absorbed by sophisticated investors recently

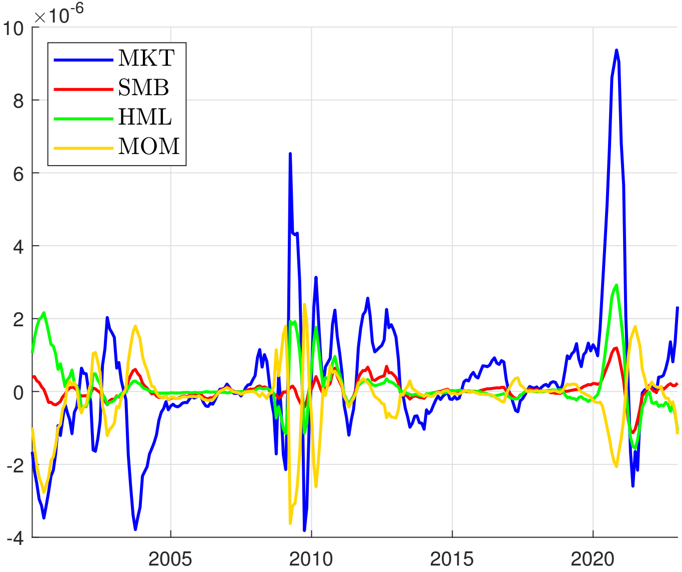
- Stock-level flows:
  $\$\text{flow}_{i,t}^{\text{stock}} = -$ mutual fund flow-induced trading of stock i at month t
  $\$\text{flow}^{\text{stock}} \uparrow$: retail selling or sophisticated buying
- Aggregate stock-level flow to factor-level

$$\text{flow}_{k,t}^{\text{factor}} := \sum_{\text{stock } i} \$\text{flow}_{i,t}^{\text{stock}} \, \text{cov}_{i,k,t}$$

↑ stock's exposure to factor k

- Accumulate flow in recent six months, with normalization

Construction result: $\widetilde{q}_{k,t}$ time-series plot



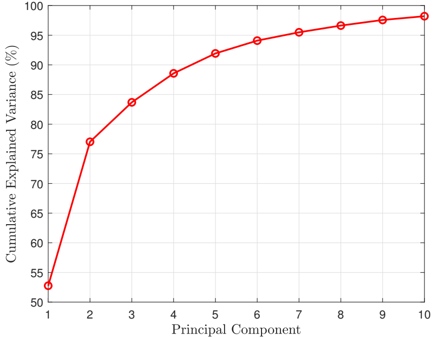
$q's$ are not highly correlated across factors

robust evidence across different factors

Correlation matrix for q's of FFC4

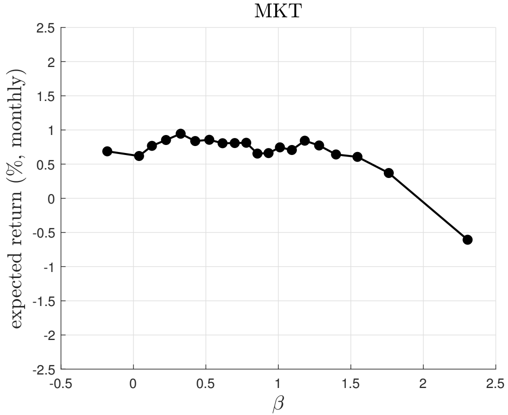
	MKT	SMB	HML	MOM
MKT	1			
SMB	0.55	1		
HML	0.47	0.57	1	
MOM	-0.47	-0.23	-0.75	1

PC variances for q's of 153 JKP factors



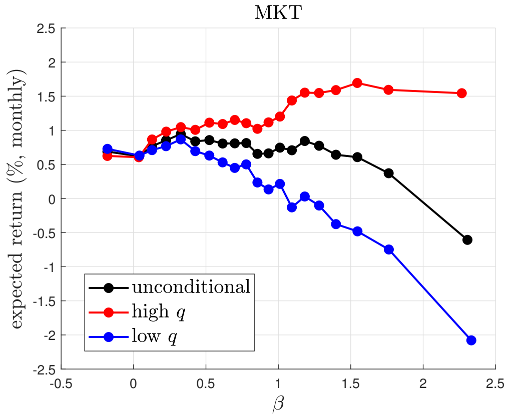
Baseline: security market line (SML) is flat

contradicts "high risk, high return"



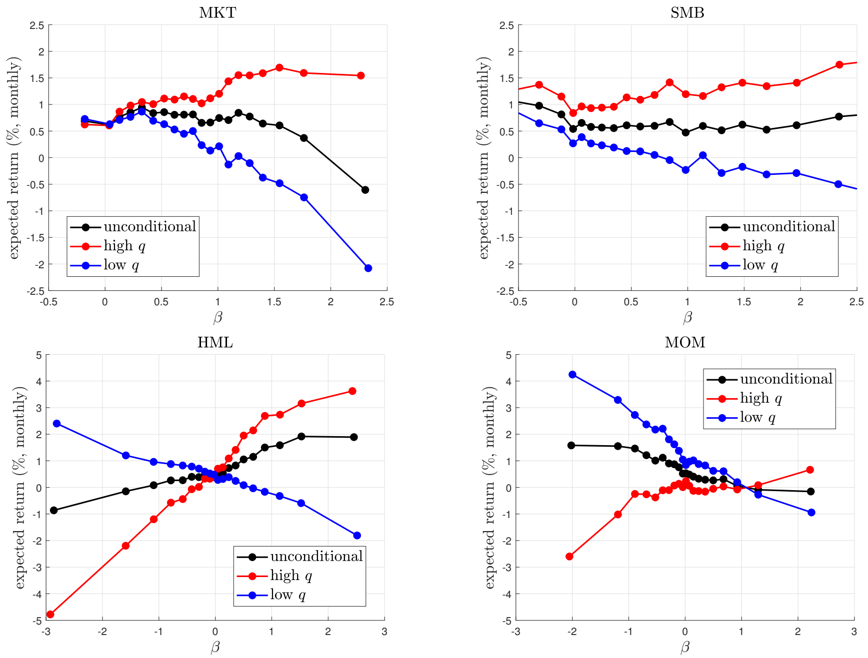
SML as non-parametric regression: $\mathbb{E}_t[r_{i,t+1}] = Er(\beta_{i,k,t})$ for the stock-month panel

Risk-return tradeoff (SML) conditioning on q



SML as non-parametric regression: $\mathbb{E}_t[r_{i,t+1}] = Er(\beta_{i,k,t})$

upgraded SML: one more input: $\mathbb{E}_t[r_{i,t+1}] = Er(\beta_{i,k,t}, q_{k,t})$



MKT
SMB
HML
MOM
expected return (%, monthly)
β
unconditional
high q
low q

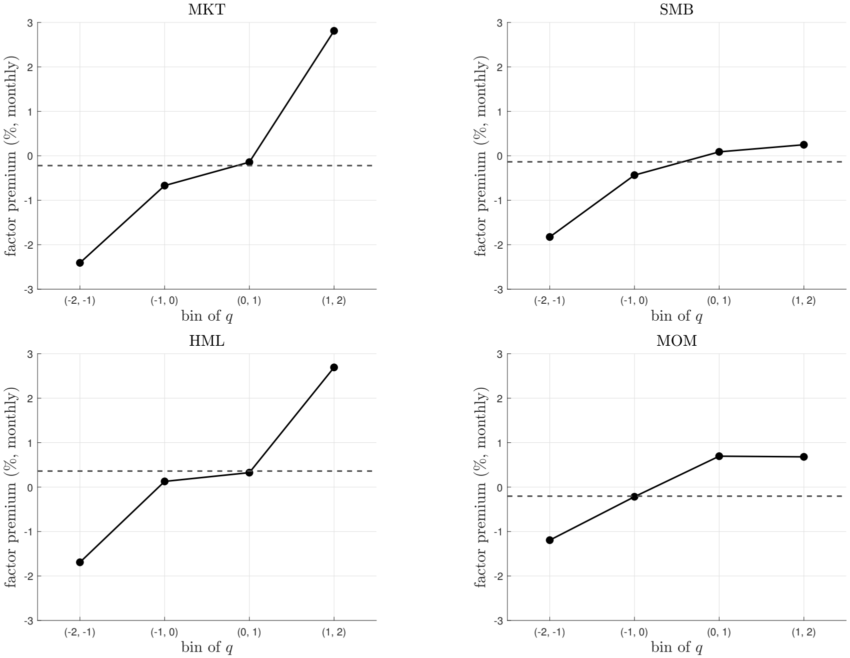
Fama-MacBeth factor premium increases with $q_{k,t}$

Estimation:

- Fama-MacBeth: cross-sectional reg $r_{i,t+1}$ on $\widehat{\beta}_{k,i,t}$, get coef. $\gamma_{k,t+1}$
  - Canonical: $\mu_k =$ time-series average of $\gamma_{k,t+1}$
  - Upgraded: varying $\mu_{k,t} = \mu_k(q_{k,t})$ conditional on $q_{k,t}$

Model:

$$\mathbb{E}_t[r_{i,t+1}] = Er(\beta_{i,k,t}, q_{k,t}) = \beta_{i,k,t}\mu_k(q_{k,t})$$


MKT
SMB
HML
MOM
factor premium (%, monthly)
bin of q
(-2, -1)
(-1, 0)
(0, 1)
(1, 2)
3
2
1
0
-1
-2
-3

BTQ (beta-times-quantity) predicts stock returns

- Factor pricing (APT):

$$\mathbb{E}_t[r_{i,t+1}] = \sum_k \beta_{i,k}\mu_{k,t}$$

- Factor premium is constant vs. linear function of $q_{k,t}$:

$$\mu_{k,t} = \mu_k \quad \text{vs.} \quad \lambda_k q_{k,t}$$

- Plug in:

$$\mathbb{E}_t[r_{i,t+1}] = \sum_k \mu_k \beta_{i,k,t} \quad \text{vs.} \quad \sum_k \lambda_k \beta_{i,k,t} q_{k,t}$$

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- Estimation: BTQ predictive regression (stock-month panel)

$$r_{i,t+1} = \sum_k \lambda_k \left(\widehat{\beta}_{i,k,t} q_{k,t}\right) + error_{i,t+1}, \qquad \forall i,t$$

vs. "β-only"

$$r_{i,t+1} = \sum_k \mu_k \widehat{\beta}_{i,k,t} + error_{i,t+1}, \qquad \forall i,t$$

BTQ vs. β-only, single factor

	Fama-French-Carhart factors				Across 153 JKP factors		
	MKT	SMB	HML	MOM	Q25	Median	Q75
Panel A: IS R^2 comparison, full sample 2000-2022 (%)							
BTQ	1.01	0.30	1.00	0.91	0.39	0.62	0.95
β-only	0.05	0.05	0.12	0.06	0.02	0.06	0.10
Panel B: OOS R^2 comparison, evaluation window 2010-2022 (%)							
BTQ	0.75	0.60	0.84	0.65	0.20	0.38	0.67
β-only	0.05	-0.10	0.15	0.02	-0.03	0.04	0.11

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BTQ vs. β-only, single factor, coefficients

	Fama-French-Carhart factors				Across 153 JKP factors		
	MKT	SMB	HML	MOM	Q25	Median	Q75
Panel C: full-sample coefficient comparison: 2000-2022							
BTQ							
λ_k	1.80	0.72	1.48	1.77	0.62	0.99	1.48
t-stat	(4.18)	(2.76)	(3.52)	(3.38)	(2.24)	(2.96)	(3.69)
β-only							
μ_k	0.38	0.31	0.56	-0.50	-0.33	-0.14	0.22
t-stat	(1.07)	(1.25)	(1.71)	(-1.23)	(-1.52)	(-0.71)	(1.11)

BTQ vs. β-only, multi-factor

	CAPM	FF3	FF3C	FF5	FF5C
	$K = 1$	3	4	5	6
Panel A: IS R^2, full sample 2000-2022 (%)					
BTQ	1.01	1.17	1.19	1.17	1.21
β-only	0.05	0.17	0.21	0.18	0.22
Panel B: OOS R^2, evaluation window 2010-2022 (%)					
BTQ	0.75	1.03	1.07	0.44	0.65
β-only	0.05	0.15	0.22	-0.26	-0.05

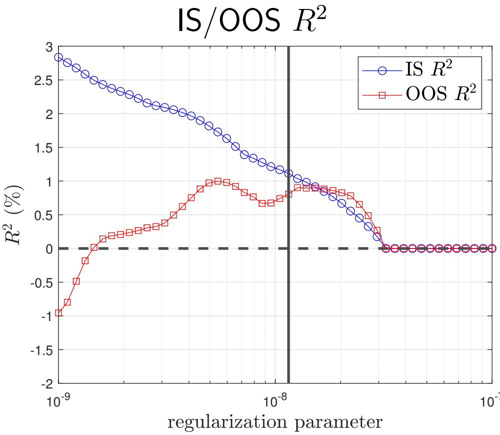
coefficents

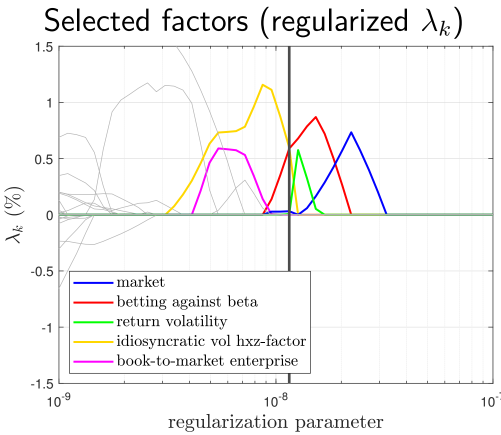
“Taming the factor zoo” with BTQ

- So many proposed factors, which are fundamental?
- New perspective to discipline factors with quantity
  - old question: $\mu_k > 0$? is there factor premium?
  - new question: $\lambda_k > 0$? does factor premium vary with investor risk holdings?
- Method:
  - BTQ prediction with 159 FF+JKP factors
  - factor selection with Lasso

BTQ, selecting from factor zoo

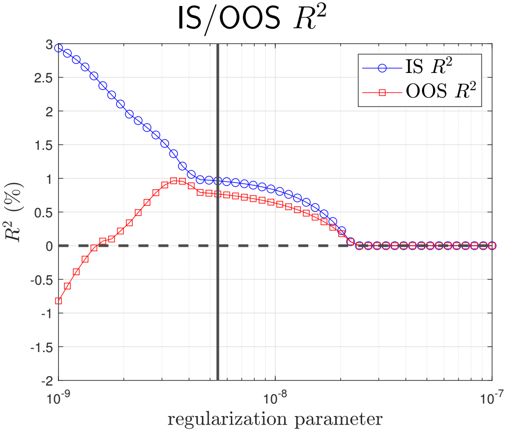
OOS predictive $R^2 \approx$ 1%, 5 factors selected, positive coefficients

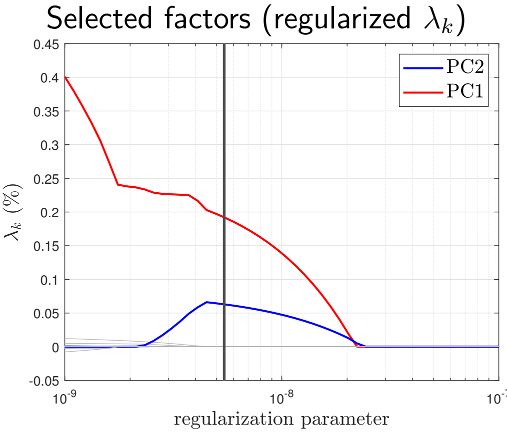




BTQ, selecting from PC factors

PC1 and PC2 selected, positive coefficients, high OOS R^2





Quantity-premium association is stronger when intermediary risk-bearing capacity is lower

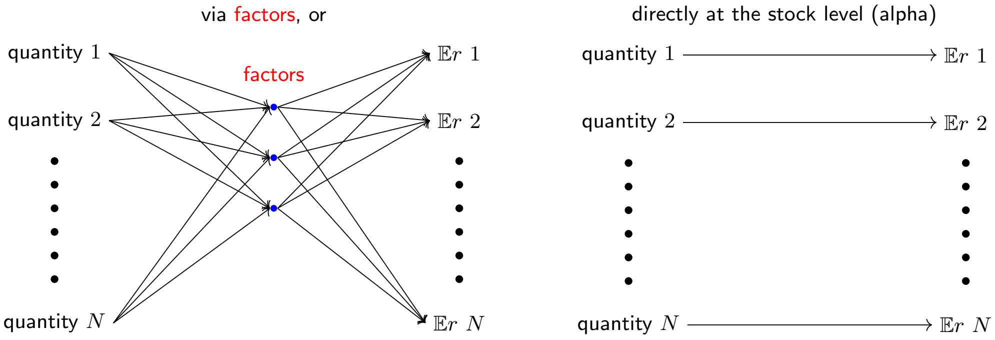
support risk-based interpretation of quantity-premium relation

$$r_{i,t+1} = \lambda_{k,\text{const}} \widehat{\beta}_{i,k,t} q_{k,t} + \lambda_{k,\text{slope}} \widehat{\beta}_{i,k,t} q_{k,t} \times \text{risk-bearing capacity}_t + error_{i,t+1}$$

	baseline BTQ	BTQ $\times$ risk-bearing capacity	
risk-bearing capacity proxy used	none	ΔICR	BKX return
A. Market factor			
$\lambda_{\text{mkt,const}}$ (%)	1.80	2.49	1.21
t-stat	(4.18)	(4.17)	(2.76)
$\lambda_{\text{mkt,slope}}$ (%)		**−1.11**	**−0.90**
t-stat		**(−2.24)**	**(−3.12)**
full-sample R^2 (%)	1.01	1.21	1.37
OOS R^2 (%)	0.75	0.62	0.80

Alpha model with quantity at individual stock level

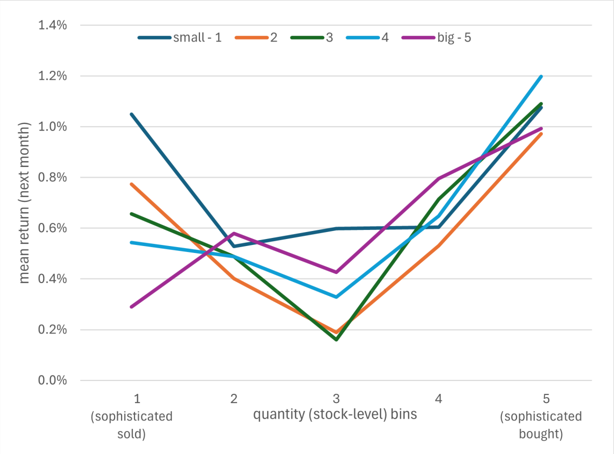
- Quantity affects expected returns via factors or also directly at stock level?



- Yes, quantity-driven alpha complements BTQ
- U-shaped quantity$_{i,t}$-$\mathbb{E}_t r_{i,t+1}$ relation, mostly in small stocks

Alpha model with quantity at individual stock level
(preliminary results)

- U-shaped quantity$_{i,t}$-$\mathbb{E}_t r_{i,t+1}$ relation, mostly in small stocks
  potential trend-following of extreme mutual fund inflows (maybe meme stocks)
- $q_{i,t}^{stock}$–size 5×5 double sort:



More results

- $q_{\text{mkt},t}$ negatively correlated with sentiment measures
  support interpretation of q direction: $q \uparrow=$ sophisticated buy / noise sell
- β_k and q_k cannot mis-match
  a factor's q_k is only relevant to risk-return trade-off along that factor's β_k
  suggest factor risk structure is essential
- beta-times-[factor momentum] does not work
  suggest "flow chasing past performance" is not an explanation
- beta-times-[macro variables] does not work
  suggest q is not repackaging known factor return predictors

- Robust results to size groups, time periods, and alternative q construction specifications

Quantity, Risk, and Return

factor risk + quantity to explain expected stock returns

Findings:

- Risk-return tradeoff (β-$\mathbb{E}r$ relation) depends on quantity
- BTQ predicts stock returns
- A new perspective to the "factor zoo" problem with quantity

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